TEST CODE **02205020**

FORM TP 2014133

MAY/JUNE 2014

C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

APPLIED MATHEMATICS

MATHEMATICAL APPLICATIONS

UNIT 2 – Paper 02

2 hours 30 minutes

26 MAY 2014 (p.m.)

This examination paper consists of THREE sections: Discrete Mathematics, Probability and Distributions, and Particle Mechanics.

Each section consists of 2 questions.
The maximum mark for each section is 50.
The maximum mark for this examination is 150.
This examination consists of 8 printed pages and 1 answer sheet for Question 5 (a) (i).

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. DO NOT open this examination paper until instructed to do so.
2. Answer ALL questions from the THREE sections.
3. Unless otherwise stated in the question, all numerical answers MUST be given exactly OR to three significant figures as appropriate.

Examination Materials:

Mathematical formulae and tables (Revised 2010)
Electronic calculator
Ruler and graph paper

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

NOTHING HAS BEEN OMITTED.

SECTION A

MODULE 1: DISCRETE MATHEMATICS

Answer BOTH questions.

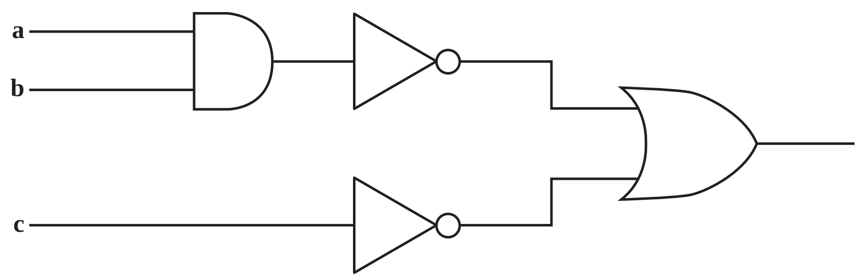
1. (a) State the contrapositive of $p \Rightarrow \sim q$. **[3 marks]**

(b) Construct a truth table for the inverse of $p \Rightarrow \sim q$. **[5 marks]**

(c) (i) Construct a truth table for $(p \rightarrow q) \vee (q \rightarrow r)$. **[5 marks]**

(ii) Hence, state with reason, whether (i) above is a tautology or a contradiction. **[2 marks]**

(d) Determine the Boolean expression for the following logic circuit.



[4 marks]

(e) (i) Draw a switching circuit for the Boolean expression $A \vee (B \wedge C)$. **[3 marks]**

(ii) Use the distributive law to expand the Boolean expression $A \vee (B \wedge C)$. **[3 marks]**

Total 25 marks

2. (a) Eight activities A, B, C, D, E, F, G and H with their preceeding activities and duration times are given in the table below.

Activity	**Duration**	**Preceding activities**
A	6	–
B	5	A
C	8	A
D	3	A
E	3	D
F	9	E
G	9	E
H	10	B, F, G, C

(i) Using the algorithm method, or otherwise, construct the activity network for these activities. **[12 marks]**

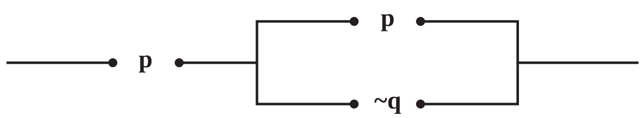
(ii) Copy and complete the following table, giving the earliest start time, latest start time and float time for EACH activity.

Activity	**Earliest Start Time**	**Latest Start Time**	**Float Time**
A			
B			
C			
D			
E			
F			
G			
H			

[4 marks]

(iii) Hence, obtain the critical path(s). **[2 marks]**

(b) (i) Represent the circuit below as a Boolean expression.



[3 marks]

(ii) Construct its truth table. **[4 marks]**

Total 25 marks

SECTION B

MODULE 2: PROBABILITY AND DISTRIBUTIONS

Answer BOTH questions.

3. (a) A and B are two independent events such that $P(A) = 0.6$, $P(B) = 0.15$.

Calculate $P(A' \cap B')$. **[4 marks]**

(b) In a choir with 30 members, 12 sing soprano, 7 sing alto, 6 sing tenor and 5 sing bass. Three members of the choir are randomly chosen to sing for a special occasion.

(i) Determine the probability that

a) two sing soprano and one sings tenor **[4 marks]**

b) one soprano, one tenor and one bass are chosen **[4 marks]**

c) three tenors are chosen given that the three persons all sing the SAME part **[5 marks]**

(ii) A committee of 9 is to be drawn from the members of the choir. Determine the probability that the committee contains EXACTLY 2 basses and 3 tenors. **[4 marks]**

(iii) The 6 tenors and 5 basses are to be seated at a circular table so that two tenors are next to each other, and the remainder sit alternately. In how many ways can this be done? **[4 marks]**

Total 25 marks

GO ON TO THE NEXT PAGE

4. (a) A cloth manufacturer knows that faults occur randomly in the production process at a rate of 3 every 15 metres.

(i) Find the probability that there are of EXACTLY 4 faults in a 15-metre length of cloth. **[3 marks]**

(ii) Calculate the probability of AT LEAST 2 faults in a 60-metre length of cloth. **[3 marks]**

(b) A crate contains oranges whose masses can be modelled by a normal distribution with mean 62.2 g and standard deviation of 3.6 g.

An orange is taken at random from the crate. Calculate the probability that the mass is

(i) less than 60 g. **[4 marks]**

(ii) between 61 g and 64 g. **[4 marks]**

(c) Two independent random variables X and Y have probability distribution functions given by

X	0	1	2
$P(X = x)$	0.2	0.3	0.5

Y	0	1	2	3	4
$P(Y = y)$	0.2	0.1	0.3	0.25	0.15

(i) Calculate $P(X + Y = 3)$. **[2 marks]**

(ii) Evaluate:

a) $E(X)$ **[1 mark]**

b) Var (X) **[1 mark]**

c) $E(Y)$ **[1 mark]**

d) Var (Y) **[1 mark]**

(iii) Hence, determine

a) $E(3X - 2Y)$ **[2 marks]**

b) Var $(3X - 2Y)$. **[3 marks]**

Total 25 marks

SECTION C

MODULE 3: PARTICLE MECHANICS

Answer BOTH questions.

[Take g as 10 ms^{-2}]

5. (a) A particle moves along a straight line and the origin O is a fixed point on that line. The displacement s metres of the particle from O at time t seconds is $s = (t - 2)(t - 6)$.

(i) **On the answer sheet provided as an insert**, draw a displacement time graph for $0 \leq t \leq 8$. **[4 marks]**

(ii) From your graph calculate

a) the total distance travelled in the period $0 \leq t \leq 5$ **[3 marks]**

b) the average velocity over the period $0 \leq t \leq 5$ **[3 marks]**

c) the time at which the velocity is zero. **[2 marks]**

(b) A particle of mass m kg rests on a horizontal plane such that the resultant S of the normal force and the frictional force makes an angle of λ with the normal. A force P inclined at an angle α to the plane is applied to the particle until it is just about to move.

(i) Draw a force diagram to illustrate this information. **[4 marks]**

(ii) Find the LEAST value of P and the value of α when P is least. **[7 marks]**

(iii) Determine the LEAST value of P in terms of m when $\alpha = 30°$. **[2 marks]**

Total 25 marks

6. (a) Formulate the equation of the trajectory of a projectile. **[4 marks]**

(b) A ball is projected with velocity 45 ms^{-1} at an angle of inclination α to the horizontal from a point A, which is at a height 4 metres above the horizontal ground. The ball strikes the ground at B, which is at a horizontal distance of 90 metres from the point A. Ignoring air resistance, answer the following:

(i) Show that $20 \tan^2 \alpha - 90 \tan \alpha + 16 = 0$. **[2 marks]**

(ii) Hence, find **to the nearest degree**, the TWO possible values of α. **[5 marks]**

(iii) Find, **to the nearest second**, the MINIMUM possible time of flight from A to B. **[3 marks]**

(c) A particle, P, moves on the x-axis. The acceleration of P at time, t seconds, $t \geq 0$, is $(3t + 5)$ ms^{-2} in the positive x-direction, When $t = 0$, the velocity of P is 2 ms^{-1} in the positive x-direction. When $t = T$, the velocity of P is 6 ms^{-1} in the positive x-direction.

Find the value of T. **[6 marks]**

(d) A particle of mass m kg slides from rest down a plane inclined at 35° to the horizontal. If the resistance to motion is ms newtons where s metres is the displacement of the particle from its initial position, find the velocity of the particle when $s = 3$. **[5 marks]**

Total 25 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.

TEST CODE **02205020**

FORM TP 2014133

MAY/JUNE 2014

C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

APPLIED MATHEMATICS

STATISTICAL ANALYSIS

UNIT 2 – Paper 02

Graph Sheet for Question 5 (a) (i)   **Candidate Number ....**

ATTACH THIS ANSWER SHEET TO YOUR ANSWER BOOKLET